$\Rightarrow \log(1) + \log(1) - 1 + 1 = C$

$0 + 0 - \cancel{1} + \cancel{1} = C$

$\Rightarrow \boxed{C = 0}$

① $\Rightarrow \log y + \log x - x + y = 0$

$\Rightarrow \log xy = x - y$

$\Rightarrow xy = e^{x-y}$ which is the equation of curve

8. Find the particular sol^n of the differential eq^n.

$x(x^2-1)\frac{dy}{dx} = 1$; $y = 0$ when $x = 2$

Sol^n:- we have. $x(x^2-1)\frac{dy}{dx} = 1$

$\Rightarrow dy = \frac{1}{x(x^2-1)} dx.$

$\Rightarrow \int dy = \int \frac{dx}{x(x^2-1)} + C$

$\Rightarrow \int dy = \int \left(\frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)} \right) dx + C$

$\Rightarrow y = -\log x + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + C$ ——— ①

$y = 0, \; x = 2$

① $\Rightarrow$

$0 = -\log 2 + \frac{1}{2}\log(2-1) + \frac{1}{2}\log(2+1) + C$

$= -\log 2 + \frac{1}{2}\log 3 + C$

$\Rightarrow \log 2 - \log\sqrt{3} = C$

$\frac{1}{x(x^2-1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$1 = A(x-1)(x+1) + Bx(x+1) + C(x-1)x$

put $x = 0$

$1 = A(-1)(1) \Rightarrow \boxed{A = -1}$

$x = 1$

$1 = B(1)(1+1) \Rightarrow \boxed{B = \frac{1}{2}}$

$x = -1$

$1 = C(-1)(-1-1) \Rightarrow \boxed{C = \frac{1}{2}}$

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$$\Rightarrow \boxed{\log \frac{2}{\sqrt{3}} = c}$$ (8)

Putting the value of c in (1)

$$y = -\log(x) + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + \log\frac{2}{\sqrt{3}}$$

$$y = -\log x + \frac{1}{2}\log(x^2-1) + \log\frac{2}{\sqrt{3}}$$

which is the required sol^n.

Equations Reducible to variable separable Form

Differential Equation of the form

$\frac{dy}{dx} = f(ax+by+c)$ can be reduced to variable separable form by putting

$ax+by+c = t$

Separate the variable and integrate

In solution put $t = ax+by+c$

Ex. 1 Solve :- $\frac{dy}{dx} = (4x+y+1)^2$

Sol^n:- We have $\frac{dy}{dx} = (4x+y+1)^2$ — ①

put $4x+y+1 = t$ — ②

$\Rightarrow 4 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \boxed{\frac{dy}{dx} = \frac{dt}{dx} - 4}$ — ③

By using ②, ③ in ①, we get

$\frac{dt}{dx} - 4 = t^2 \Rightarrow \frac{dt}{dx} = t^2 + 4$

$\Rightarrow \frac{dt}{t^2+4} = dx$

Integrating both sides.

$\Rightarrow \int \frac{dt}{t^2+2^2} = \int dx + C$

$\Rightarrow \frac{1}{2}\operatorname{Tan}^{-1}\frac{t}{2} = x + c$

$\Rightarrow \operatorname{Tan}^{-1}\frac{t}{2} = 2(x+c) \Rightarrow \frac{t}{2} = \operatorname{Tan}2(x+c)$

$\Rightarrow t = 2\operatorname{Tan}2(x+c)$

$\therefore 4x+y+1 = 2\operatorname{Tan}2(x+c)$

which is the required sol^n.

Ex. 2 :- Solve : $\frac{dy}{dx} = 1 + e^{x-y}$

sol^n:- We have $\frac{dy}{dx} = 1 + e^{x-y}$ ——— (*)

put $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$.

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$.

(*) $\Rightarrow \cancel{1} - \frac{dt}{dx} = \cancel{1} + e^{t}$

$\Rightarrow \frac{dt}{dx} = -e^{t} \Rightarrow \frac{dt}{-e^{t}} = dx$

Integrating both sides, we get

$\Rightarrow \int -e^{-t}\,dt = \int dx + c$

$\Rightarrow e^{t} = x + c \quad \therefore \boxed{e^{x-y} = x + c}$

Ex. 3 solve the differential eq^n

$$\frac{dy}{dx} = \cos(x+y+2) \text{ given that } y(0) = -2$$

or $x=0$, $y=-2$

Sol^n:- we have $\frac{dy}{dx} = \cos(x+y+2)$ —— (A)

put $x+y+2 = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$.

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{dt}{dx} - 1}$$

(A) $\Rightarrow \frac{dt}{dx} - 1 = \cos t \Rightarrow \frac{dt}{dx} = \cos t + 1$

$$\Rightarrow \frac{dt}{\cos t + 1} = dx.$$

Integrating both sides, we get

$$\Rightarrow \int \frac{dt}{2\cos^2\frac{t}{2}} = \int dx + C$$

$\because \cos 2x = \cos^2 x - \sin^2 x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$\Rightarrow \int \frac{1}{2} \sec^2\frac{t}{2}\, dt = \int dx + C$$

$$\Rightarrow \frac{\frac{1}{2}\tan\frac{t}{2}}{\frac{1}{2}} = x + C \Rightarrow \tan\frac{t}{2} = x + C$$

$$\Rightarrow \tan\left(\frac{x+y+2}{2}\right) = x + C \quad \text{—— (1)}$$

put $x=0$, $y=-2$ in eq^n (1)

$\Rightarrow \tan\left(\frac{0+2+2}{2}\right) = 0 + c$

$\Rightarrow \tan(0) = c \Rightarrow \boxed{c = 0}$

Hence required sol^n is $\tan\left(\frac{x+y+2}{2}\right) = x$

Ex.:- $\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$

Sol^n:- we have $\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$

$$\Rightarrow \frac{dy}{dx} = \sin(x+y) \quad \text{——— (1)}$$

put $x+y=t \Rightarrow 1+\frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{dt}{dx} - 1}$$

(1) $\Rightarrow \frac{dt}{dx} - 1 = \sin t \Rightarrow \frac{dt}{dx} = 1+\sin t$

$$\Rightarrow \frac{dt}{1+\sin t} = dx$$

$$\Rightarrow \int \frac{dt}{1+\sin t} = \int dx + c$$

Try your self

$\left(\tan(x+y) - \sec(x+y) = x+c\right)$